

Viability of the exact tri-bimaximal mixing at M_{GUT} in $SO(10)$

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Abstract

General structures of the charged lepton and the neutrino mixing matrices leading to tri-bimaximal leptonic mixing are determined. These are then integrated into an $SO(10)$ model within which detailed fits to fermion masses and mixing angles are given. It is shown that one can obtain excellent fits to all the fermion masses and quark mixing angles keeping tri-bimaximal leptonic mixing intact. Different perturbations to the basic structure are considered and those which can or which cannot account for the recent T2K and MINOS results on the reactor mixing angle θ_{13}^l are identified.

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I. INTRODUCTION

The Tri-bimaximal (TBM) leptonic mixing [1] provides an important clue in search of possible flavour structure [2] governing the leptonic masses and mixing angles. It predicts $\sin^2 \theta_{12}^l = 1/3$ and $\sin^2 \theta_{23}^l = 1/2$ respectively for the solar and the atmospheric mixing angles both of which agree nearly within 1σ with the latest global analysis [3, 4] of the neutrino oscillation data. TBM also predicts vanishing reactor mixing angle θ_{13}^l . This can be reconciled with the latest T2K [5] (MINOS [6]) results at 2.5σ (1.6σ) and with the global analysis [3, 4] at about 3σ . This suggests that the TBM may be a good zeroth order approximation which needs perturbations affecting mainly the reactor mixing angle θ_{13}^l . While such perturbations may arise from some underlying flavor symmetry, see [7] for examples, it would be more appropriate if some independent mechanism like grand unified theory (GUT) [8] governs these perturbations. We wish to analyze here the TBM structure and perturbations to it within a grand unified model based on $SO(10)$ gauge symmetry.

Incorporating the tri-bimaximal mixing into GUTs, particularly based on the $SO(10)$ gauge group is quite challenging, see [9] for some examples. Since all fermions in a given generation are unified into a single 16 dimensional irreducible representation of $SO(10)$, imposition of the TBM structure on the leptonic mass matrices also constrains the quark mass matrices. It is not clear if the requirement of the exact tri-bimaximal mixing among leptons would be consistent with a precise description of the quark masses and mixing. We suggest a general method of incorporating the exact TBM structure within $SO(10)$ and use it to obtain quantitative description of the fermion masses and mixing.

Our approach is purely phenomenological. We do not use any flavour symmetry but determine the most general structure of the leptonic mixing matrices required for obtaining the tri-bimaximal mixing. We then try to integrate this structure into $SO(10)$ and discuss numerical fits to the quark masses and mixing leaving the TBM intact. Then we discuss perturbations to the basic structure and their effects on observables.

II. LEPTONIC MIXING MATRICES AND TBM

We shall derive general forms for the neutrino mass matrix M_ν and the left handed charged lepton mixing matrix U_l which lead in the flavour basis to a neutrino mass matrix $\mathcal{M}_{\nu f}$ exhibiting the TBM structure. We define the TBM structure for $\mathcal{M}_{\nu f}$ as follows:

$$\mathcal{M}_{\nu f} = \frac{1}{3} \begin{pmatrix} 2f_1 + f_2 & f_2 - f_1 & f_2 - f_1 \\ f_2 - f_1 & \frac{1}{2}(f_1 + 2f_2 + 3f_3) & \frac{1}{2}(f_1 + 2f_2 - 3f_3) \\ f_2 - f_1 & \frac{1}{2}(f_1 + 2f_2 - 3f_3) & \frac{1}{2}(f_1 + 2f_2 + 3f_3) \end{pmatrix}, \quad (1)$$

where $f_{1,2,3}$ are complex neutrino masses. This matrix is diagonalized by

$$U_{PMNS} = O_{TBM} Q, \quad (2)$$

where Q is a diagonal phase matrix and

$$O_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (3)$$

A more general definition of TBM structure would be to replace $\mathcal{M}_{\nu f}$ and U_{PMNS} above by $P_l \mathcal{M}_{\nu f} P_l$ and $P_l^* U_{PMNS}$, where P_l denotes a diagonal phase matrix. Since P_l can be rotated away by redefining the charged lepton fields, we shall refer to TBM structure as the one defined by Eqs. (1,2).

It is known [10] that $\mathcal{M}_{\nu f}$ in Eq. (1) is invariant under a $Z_2 \times Z_2$ symmetry. The elements of the $Z_2 \times Z_2$ are defined as

$$S_2 = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad \text{and} \quad S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (4)$$

and satisfy

$$S_{2,3}^T \mathcal{M}_{\nu f} S_{2,3} = \mathcal{M}_{\nu f} \quad (5)$$

We shall exploit this $Z_2 \times Z_2$ invariance in arriving at the structure of U_l . As noted in [11], one can always choose a specific basis in which M_ν exhibits the TBM structure and is thus invariant under $Z_2 \times Z_2$:

$$S_{2,3}^T M_\nu S_{2,3} = M_\nu \quad (6)$$

If M'_ν in an arbitrary basis is not invariant under $Z_2 \times Z_2$ then one can go to a new basis with $M_\nu = U^T M'_\nu U$ and choose U in such a way that

$$U_\nu^T M_\nu U_\nu = D_\nu, \quad (7)$$

with D_ν a diagonal matrix with real positive elements and

$$U_\nu \equiv O_{TBM} P. \quad (8)$$

P being a general diagonal phase matrix. Let U_l denote the mixing matrix among the left handed charged leptons in a basis in which M_ν is $Z_2 \times Z_2$ symmetric. If such a defined U_l itself is $Z_2 \times Z_2$ symmetric, *i.e.* satisfies

$$S_{2,3}^T U_l S_{2,3} = U_l \quad (9)$$

then $\mathcal{M}_{\nu f}$ will also satisfy Eq. (5) and thus would exhibit the TBM structure of Eq. (1). This follows trivially from the definition

$$\mathcal{M}_{\nu f} = U_l^T M_\nu U_l, \quad (10)$$

after using Eqs. (6,9). Thus the $Z_2 \times Z_2$ invariance of U_l is sufficient to ensure the TBM for $\mathcal{M}_{\nu f}$.

Above equation allows us to determine TBM preserving class of U_l in a basis with M_ν satisfying Eq. (6). S_3 invariance corresponds to imposing the μ - τ interchange symmetry on U_l . The S_2 invariance further requires $U_l^T = U_l$ and that the sum of elements in each of its row must be equal. Such a U_l can be parameterized as

$$U_l = e^{i\alpha} P_l \tilde{U}_l P_l, \quad (11)$$

where $P_l = \text{diag.}(1, e^{i\beta}, e^{i\beta})$ is a diagonal phase matrix and

$$\tilde{U}_l = \begin{pmatrix} c_\theta & \frac{s_\theta}{\sqrt{2}} & \frac{s_\theta}{\sqrt{2}} \\ \frac{s_\theta}{\sqrt{2}} & -\frac{1}{2}(c_\theta + e^{i\delta}) & -\frac{1}{2}(c_\theta - e^{i\delta}) \\ \frac{s_\theta}{\sqrt{2}} & -\frac{1}{2}(c_\theta - e^{i\delta}) & -\frac{1}{2}(c_\theta + e^{i\delta}) \end{pmatrix}, \quad (12)$$

with $\tan \theta = -2\sqrt{2} \cos \beta$. U_l is thus fully determined by three phase angles α, β and δ .

The form of U_l as given above is the most general one required in order to obtain TBM in the basis with M_ν satisfying Eq. (6). The generality is proved by noticing that the $Z_2 \times Z_2$ invariance of U_l is also necessary if $\mathcal{M}_{\nu f}$ is to exhibit the TBM structure. This follows in a straightforward manner. Assume that $\mathcal{M}_{\nu f}$ has TBM structure of Eq. (1). The U_{PMNS} matrix in this case can be chosen to have the form in Eq. (2). Since $U_l = U_\nu U_{PMNS}^\dagger$, it has the following form in the basis specified by Eq. (8):

$$\begin{aligned} U_l &= O_{TBM} P Q^* O_{TBM}^T, \\ &= \frac{1}{3} \begin{pmatrix} 2p_1 + p_2 & p_2 - p_1 & p_2 - p_1 \\ p_2 - p_1 & \frac{1}{2}(p_1 + 2p_2 + 3p_3) & \frac{1}{2}(p_1 + 2p_2 - 3p_3) \\ p_2 - p_1 & \frac{1}{2}(p_1 + 2p_2 - 3p_3) & \frac{1}{2}(p_1 + 2p_2 + 3p_3) \end{pmatrix}, \end{aligned} \quad (13)$$

where p_i denote the elements of the diagonal phase matrix PQ^* . Interestingly, the above U_l is obtained from the general TBM $\mathcal{M}_{\nu f}$ Eq. (1), by replacing the neutrino masses with the phases p_i and like $\mathcal{M}_{\nu f}$ such a U_l is automatically $Z_2 \times Z_2$ symmetric. Thus Eq. (9) also becomes necessary for the $Z_2 \times Z_2$ invariance of $\mathcal{M}_{\nu f}$. Eq. (13) provides an alternative parametrization of U_l . It reduces to the earlier parametrization in Eq. (12) with the definition

$$\begin{aligned} p_1 &= -e^{i(\alpha+\beta-\eta)}, \\ p_2 &= e^{i(\alpha+\beta+\eta)}, \\ p_3 &= -e^{i(\alpha+2\beta+\delta)}, \end{aligned} \quad (14)$$

with $\cos \eta = -3c_\theta \cos \beta$ and $\sin \eta = -c_\theta \sin \beta$.

We note that the U_ν and U_l in Eqs. (8,11) are defined up to a simultaneous redefinition $U_\nu \rightarrow U U_\nu$ and $U_l \rightarrow U U_l$. In addition, U_l can also be multiplied by an arbitrary phase matrix from the right. Since P_l is arbitrary, the U_l in some model may not appear to have the $Z_2 \times Z_2$ invariance even in a basis with U_ν chosen as in Eq. (8). But the above exercise shows that U_l can always be chosen to have the TBM form by appropriate rephasing of the charged lepton fields.

The above reasoning can be applied to more general patterns of mixing and not just to TBM. The key role in this construction is played by the fact that one can always choose a basis in which M_ν is $Z_2 \times Z_2$ symmetric. This follows from the fact that the $Z_2 \times Z_2$ symmetry does not put any restrictions on the neutrino masses but only on the structure of mixing. As long as the neutrino mass matrices obey such “mass-independent” symmetries, the above construction of determining the most general U_l can be carried through. One can indeed define [10] an appropriate $Z_2 \times Z_2$ symmetry corresponding to every mixing pattern and then impose this symmetry on U_l to obtain the desired mixing structure in the flavour basis. As an example, in case of the μ - τ symmetry, one can always choose a basis in which M_ν is μ - τ symmetric. Then requiring that U_l also be μ - τ symmetric, we arrive at a general forms for U_l and M_ν which lead to a μ - τ symmetric $\mathcal{M}_{\nu f}$:

$$M_\nu = \begin{pmatrix} X & A & A \\ A & B & C \\ A & C & B \end{pmatrix} ; \quad U_l = e^{i\alpha} P_{lL} \tilde{U}_l P_{lR} \quad (15)$$

where $P_{lL,R} = \text{diag.}(1, e^{i\beta_{L,R}}, e^{i\beta_{L,R}})$ and \tilde{U}_l has the same form of Eq. (12) but now θ is an independent parameter and is not a function of β as before.

III. $SO(10)$ MODEL AND TBM

We now integrate the above leptonic structures into an $SO(10)$ model. We make the following assumptions which lead to simplification and allows us to obtain quantitative description. We (1) consider a supersymmetric $SO(10)$ model with the Higgs transforming as 10, $\overline{126}$, 120 representations of $SO(10)$ (2) impose the generalized parity as in [11–15] leading to Hermitian mass matrices and (3) assume that the dominant contribution to M_ν is a type-II seesaw, *i.e.* linear in the $\overline{126}$ Yukawa coupling. The last assumption with its attractive consequences is made in a number of $SO(10)$ models [11, 12, 16, 17]. The type-II dominance may be achieved by adding more Higgs fields. The type-I contribution can be suppressed by pushing the $B - L$ breaking scale high while the type -II contribution can be relatively enhanced if the Higgs triplet remains below the GUT scale. The latter spoils the gauge coupling unification. It was pointed out in [18] that breaking $SO(10)$ to $SU(5) \times U(1)$ around 10^{17} GeV can suppress the type-I contribution and presence of 54 plet can allow a complete multiplet transforming as 15 in $SU(5)$ to remain light. One can achieve in this way the type-II dominance without sacrificing the gauge coupling unification. The same phenomena was analyzed subsequently [19] in a model with fields transforming as $54 + 45$ (instead of the standard 210) and having a 120 plet of Higgs. The fields $54 + 45$ required to achieve the type-II dominance do not effect the fermion masses and the fermion sector can be the same as considered here.

The fermion mass relations in this case after electroweak symmetry breaking can be

written in their most general forms as [13–15]:

$$\begin{aligned}
M_d &= H + F + iG , \\
M_u &= r(H + sF + it_u G) , \\
M_l &= H - 3F + it_l G , \\
M_D &= r(H - 3sF + it_D G) , \\
M_L &= r_L F , \\
M_R &= r_R^{-1} F .
\end{aligned} \tag{16}$$

where (G) H , F are real (anti)symmetric matrices. $r, s, t_l, t_u, t_D, r_L, r_R$ are dimensionless real parameters. The effective neutrino mass matrix for three light neutrinos resulting after the seesaw mechanism can be written as

$$M_\nu = r_L F - r_R M_D F^{-1} M_D^T \equiv M_\nu^{II} + M_\nu^I . \tag{17}$$

The first term proportional to F denotes type-II seesaw contribution. In the numerical analysis that follows, we shall assume that M_ν is entirely given by this term and subsequently analyze the effect of a small type-I corrections on the numerical solution found.

We can always rotate the 16-plet fermions in generation space in such a way that $M_\nu \propto F$ is diagonalized by the TBM matrix.

$$F \rightarrow R^T F R = F_{TBM} \equiv O_{TBM} \text{Diag.}(f_1, f_2, f_3) O_{TBM}^T \tag{18}$$

where f_i are now real eigenvalues of F and the O_{TBM} is given by Eq. (3). The matrix $(G)H$ maintains its (anti)symmetric form in such basis and we use the same label for them in the rotated basis. In case of type-II seesaw dominance, the light neutrino mass matrix $M_\nu = r_L F_{TBM}$ has the form given on the RHS of Eq. (1). The model has altogether 17 independent real parameters (3 in F_{TBM} , 6 in H , 3 in G , r , s , t_u , t_l and r_L) which determine the entire 22 low energy observables of the fermion mass spectrum. Some of these parameters can be fixed by the known values of observables directly. As noted in [15], the $SO(10)$ relation for the charged lepton mass matrix in Eq. (16) can be rewritten as

$$H + it_l G = V_l D_l V_l^\dagger + 3F_{TBM} , \tag{19}$$

where D_l is a diagonal charged lepton mass matrix. Since H and G are real, the real and imaginary parts of the RHS separately determine H and $t_l G$ in terms of the charged lepton masses, parameters of F_{TBM} and V_l . V_l is a unitary matrix that diagonalizes M_l and contains nine free parameters in the most general situation. One can suitably write $V_l = \tilde{V}_l P$ where P is diagonal phase matrix and \tilde{V}_l contains six real parameters. From Eq. (19), it is easy to see that the phase matrix P does not play any role in determining H and G and can be removed. So the nine real parameters of LHS can be related to six real parameters of \tilde{V}_l , three charged lepton masses and parameters of F_{TBM} in Eq. (19). This fixing helps us in numerical analysis as we will see in the next subsection.

We shall present numerical analysis in two different cases. (A) Corresponding to the most general V_l (B) with $V_l = U_l$ given as in Eqs. (11,12). The case (A) has already been studied numerically in [11, 12, 15]. We refine this analysis using a different numerical procedure and taking into account the results of the most recent global fits [3] to neutrino data. This also serves as a benchmark with which to compare the case (B) which leads to the exact TBM at M_{GUT} .

A. Numerical Analysis: The most general case

We study the viability of Eq. (16) with the experimentally observed values of fermion masses and mixing angles through numerical analysis. For this, we construct a χ^2 function defined as

$$\chi^2 = \sum_i \left(\frac{P_i - O_i}{\sigma_i} \right)^2 \quad (20)$$

where the sum runs over different observables. P_i denote the theoretical values of observables determined by the expressions given in Eq. (16) and O_i are the experimental values extrapolated to the GUT scale. σ_i denote the 1σ errors in O_i . Our choice of the input values of quark and lepton masses and quark mixing angles are the same as used in [11]. In this data set, the charged fermion masses at the GUT scale are [20] obtained from the low energy values using MSSM and $\tan \beta = 10$. We use the input values of lepton mixing angles from [3] which includes results from T2K and MINOS. The effect of the RG evolution on the quark mixing angles is known to be negligible. This is also true for the lepton mixing angles in case of the hierarchical neutrino spectrum. We assume such hierarchy in neutrino masses and therefore the input values of the quark mixing angles, CP phase and neutrino parameters we use correspond to their values at low energy. We reproduce all these input values in Table I for convenience of the reader.

GUT scale values with propagated uncertainty			
$m_d(\text{MeV})$	1.24 ± 0.41	$\Delta m_{sol}^2(\text{eV}^2)$	$(7.58 \pm 0.22) \times 10^{-5}$
$m_s(\text{MeV})$	21.7 ± 5.2	$\Delta m_{atm}^2(\text{eV}^2)$	$(2.35 \pm 0.12) \times 10^{-3}$
$m_b(\text{GeV})$	$1.06^{+0.14}_{-0.09}$	$\sin \theta_{12}^q$	0.2243 ± 0.0016
$m_u(\text{MeV})$	0.55 ± 0.25	$\sin \theta_{23}^q$	0.0351 ± 0.0013
$m_c(\text{GeV})$	0.210 ± 0.021	$\sin \theta_{13}^q$	0.0032 ± 0.0005
$m_t(\text{GeV})$	$82.4^{+30.3}_{-14.8}$	$\sin^2 \theta_{12}^l$	$0.306^{+0.018}_{-0.015}$
$m_e(\text{MeV})$	0.3585 ± 0.0003	$\sin^2 \theta_{23}^l$	$0.42^{+0.08}_{-0.03}$
$m_\mu(\text{MeV})$	75.672 ± 0.058	$\sin^2 \theta_{13}^l$	$0.021^{+0.007}_{-0.008}$
$m_\tau(\text{GeV})$	1.2922 ± 0.0013	J_{CP}	$(2.2 \pm 0.6) \times 10^{-5}$

TABLE I. Input values for quark and leptonic masses and mixing angles in the MSSM extrapolated at $M_{GUT} = 2 \times 10^{16}$ GeV for $\tan \beta = 10$ which we use in our numerical analysis.

We fit the above data to the fermion mass relations (16) predicted in the model by numerically minimizing the χ^2 function. As already mentioned above, this exercise has been done in [11] recently and a very good fit corresponding to $\chi^2_{min} = 0.127$ is found. We repeat the same analysis because of the following differences in our fitting procedure and because of the emergence of new results on θ_{13}^l [5, 6].

- Compared to other observables, the charged lepton masses are known very precisely with extremely small errors in their measurements. Instead of fitting them through χ^2 minimization, we use their central values as inputs on the RHS of Eq. (19). Because of this, our definition of the χ^2 function in Eq. (20) does not include the charged lepton masses in it.
- We also use the central value of the solar to the atmospheric mass squared difference ratio as an input and use it to fix f_3 through the following relation:

$$f_3 = f_2 \left(\frac{\Delta m_{atm}^2}{\Delta m_{sol}^2} + \left(1 - \frac{\Delta m_{atm}^2}{\Delta m_{sol}^2} \right) \left(\frac{f_1}{f_2} \right)^2 \right)^{1/2}. \quad (21)$$

After obtaining the solution, the overall scale of neutrino masses r_L at the minimum is determined by using the atmospheric scale as a normalization.

As a result of these simplifications, the χ^2 function in our approach includes only 13 observables, 6 quark masses, 3 quark mixing angles, a CKM phase and 3 lepton mixing angles. These are complex nonlinear functions of 12 real parameters (2 in F_{TBM} , 6 in V_l , r , s , t_l and t_u). This χ^2 is numerically minimized using the function minimization tool MINUIT. The results of our analysis are shown in column A in Table II.

We obtain an excellent fit corresponding to $\chi^2_{min} = 0.0061$ which is significantly better than $\chi^2_{min} = 0.127$ obtained in [11] using different procedure and different data set. Parameters obtained for the best fit solution are shown in Appendix. All the observables are fitted within the 0.05σ deviation from their central values. The solution at its minimum almost reproduces the central value of $\sin^2 \theta_{13}^l$ obtained in the global fit [3]. We also determine a parameter d_{FT} introduced in [11] which quantitatively measures the amount of fine-tuning needed in the parameters for obtained fit when compared with d_{Data} which is a similar parameter obtained from the data only. From our fit, we obtain $d_{FT} \sim 2.9 \times 10^4$ compared to $d_{Data} \sim 220$. These parameters depend on the definition of χ^2 and since we do not include the charged lepton masses -which have very small errors- in our χ^2 , both the parameters d_{FT} and d_{Data} are an order of magnitude smaller than [11]. However the ratio $d_{Data}/d_{FT} \sim 7.6 \times 10^{-3}$ obtained from our fit is almost similar to $d_{Data}/d_{FT} \sim 8 \times 10^{-3}$ obtained in [11] which shows that both of the solutions need substantial level of fine tunings in the model parameters.

	Case A		Case B1		Case B2	
Observables	Fitted value	Pull	Fitted value	Pull	Fitted value	Pull
$m_d[\text{MeV}]$	1.2339	-0.0148738	1.22098	-0.0463899	1.02686	-0.519852
$m_s[\text{MeV}]$	21.7214	0.00411949	21.9922	0.0561874	22.0058	0.058806
$m_b[\text{GeV}]$	1.06614	0.0438763	1.16345	0.738942	1.2842	1.60145
$m_u[\text{MeV}]$	0.550018	0.000073755	0.550234	0.000936368	0.550787	0.00314771
$m_c[\text{GeV}]$	0.209977	-0.00111886	0.209952	-0.00230315	0.210481	0.0229054
$m_t[\text{GeV}]$	82.5278	0.00421748	82.5855	0.00612198	81.7487	-0.0440052
$m_e[\text{MeV}]$	0.3585	—	0.3585	—	0.3585	—
$m_\mu[\text{MeV}]$	75.672	—	75.672	—	75.672	—
$m_\tau[\text{GeV}]$	1.2922	—	1.2922	—	1.2922	—
$\left(\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}\right)$	0.0323	—	0.031875	—	0.031875	—
$\sin \theta_{12}^q$	0.224299	-0.000688878	0.2243	0.0002182	0.224303	0.0019076
$\sin \theta_{23}^q$	0.0351032	0.00246952	0.0350951	-0.0038047	0.0351294	0.022597
$\sin \theta_{13}^q$	0.00320513	0.0102511	0.00319436	-0.0112796	0.0031749	-0.0502087
$\sin^2 \theta_{12}^l$	0.306119	0.00660722	0.3333	—	0.3333	—
$\sin^2 \theta_{23}^l$	0.418475	-0.0508353	0.5	—	0.5	—
$\sin^2 \theta_{13}^l$	0.0207708	-0.0286467	0	—	0	—
J_{CP}	2.19×10^{-5}	-0.0183401	2.21×10^{-5}	0.0194165	2.25×10^{-5}	0.0845729
δ_{MNS}	282.396	—	—	—	—	—
α_1	142.975	—	160.829	—	180	—
α_2	22.4851	—	318.593	—	0	—
r_L	6.89×10^{-10}	—	9.82×10^{-10}	—	3.53×10^{-9}	—
χ_{min}^2		0.0061		0.5519		2.8510
d_{FT}		28751		88546		31171
d_{Data}		220		200		200

TABLE II. Best fit solutions for fermion masses and mixing obtained assuming type-II seesaw dominance in the SUSY $SO(10)$ model with $10 + \overline{126} + 120$ Higgs. Various observables and their pulls at the minimum are shown for three different cases correspond to (A) the general (non TBM) leptonic mixing, (B1) Exact TBM leptonic mixing with U_l of Eqs. (11,12) and (B2) Exact TBM leptonic mixing with diagonal M_l and $t_l = 0$ (See the discussions in the text for more details). The predictions of different approaches are shown in boldface.

B. Numerical Analysis: Exact TBM

After discussion of the above general case, we now specialize to the case of the exact TBM. This case is of considerable theoretical interest since it can point to some underlying symmetry existing at M_{GUT} . We can implement the exact TBM in a model independent way

by choosing $V_l = U_l$ in Eq. (19). With this choice, all the leptonic mixing angles get fixed to their TBM values. Also the central value of the ratio of the solar to the atmospheric (mass)² differences is used as input and a parameter r_L is determined at the minimum by using the atmospheric scale. Thus the χ^2 function in Eq. (20) now involves only observables in the quark sector. As already discussed, U_l in Eq. (11,12) is parameterized by three phase angles α, β and δ . An overall phase α is irrelevant for the physical observables and can be removed. This leaves only 8 real parameters (2 in F_{TBM} , 2 in U_l , r , s , t_l and t_u) which are fitted to the 10 observables in the quark sector by minimizing the χ^2 . The results are shown in column B1 in Table II. The obtained fit corresponds to $\chi_{min}^2 = 0.552$ ($\chi_{min}^2/\text{d.o.f.} = 0.276$). Only the fitted value of m_b deviates slightly from the central value with a 0.74σ pull. All the remaining observables are fitted within 0.06σ . A set of parameters obtained for this solutions are shown in Appendix. The fit obtained here is not significantly different from the general case discussed before showing that all the fermion masses and mixing angles can be nicely reproduced along with the exact TBM within the $SO(10)$ framework discussed here.

Before we discuss possible perturbations in TBM pattern, let us discuss a very special case corresponding to a diagonal M_l . This corresponds to U_l coinciding with an identity matrix and is a special case of Eq. (11,12) with $\beta = \frac{\pi}{2}$ and $\delta = 0$. Since M_l is real and diagonal in this case, $t_l G$ must vanish in Eq. (19). If $G = 0$ then the quark mass matrices also become real and there is no room for CP violation. The viable scenario must therefore have nonzero G and hence $t_l = 0$. As a result, unlike before, the three parameters in G do not get determined from M_l , see, Eq. (19) and remain free. They can be fitted from the quark sector observables. We carried out a separate numerical analysis for this particular case and the results are shown in column B2 in Table II. The fit obtained gives relatively large $\chi_{min}^2 = 2.85$ ($\chi_{min}^2/\text{d.o.f.} = 1.43$) with more than 1σ deviation in the bottom quark mass. Although the obtained χ_{min}^2 is statistically acceptable at 90% confidence level, it is not as good as the previous one and we shall not consider this case with $U_l = I$ any further.

IV. PERTURBED TBM

The TBM is an ideal situation and various perturbations to this can arise in the model. We need to analyze these perturbations in order to distinguish this case from the generic case without the built in TBM. A deviation from tri-bimaximality can arise due to

1. renormalization group evolution (RGE) from M_{GUT} to M_Z .
2. small contribution from the sub dominant type-I seesaw term in Eq. (17).
3. the breaking of the $Z_2 \times Z_2$ symmetry in U_l which ensured TBM.

The effect of (1) is known to be negligible [21] in case of the hierarchical neutrino mass spectrum which we obtain here. We quantitatively discuss the implications of the other two scenarios via detailed numerical analysis in the following subsections.

A. Perturbation from type-I seesaw

Depending on the GUT symmetry breaking pattern and parameters in the superpotential of the theory, a type-I seesaw contribution can be dominant or sub dominant compared to type-II but it is always present and can generate deviations in an exact TBM mixing pattern in general. In the approach pursued here it is assumed that such contribution remains sub dominant and generates a small perturbation in dominant type-II spectrum. Eq. (17) can be rewritten as

$$M_\nu = r_L(F - \xi M_D F^{-1} M_D^T) \quad (22)$$

where $\xi = r_R/r_L$ determine the relative contribution of type-I term in the neutrino mass matrix.

The second term in Eq. (17) brings in two new parameters ξ and t_D present in the definition of M_D in Eq. (16). These parameters however affect only the neutrino sector. We isolate the effect of type-I contribution by choosing other parameters at the χ^2 minimum found in Section III-B. ξ and t_D remain unconstrained at this minimum and their values do not change the χ^2 obtained earlier since the latter contains only the observables in the quark sector. The ξ, t_D however generate departure from the exact TBM. We randomly vary the parameters ξ and t_D and evaluate the neutrino masses and mixing angles. While doing this, we take care that all these observables remain within their present 3σ [3] limits. Such constraints allow very small values of $|\xi| \leq 10^{-7}$. The correlations between different leptonic mixing angles found from such analysis are shown in Fig. (1).

It is seen from Fig. (1) that the perturbation induced by type-I term cannot generate considerable deviation in the reactor angle if the other two mixing angles are to remain within their 3σ range. In particular, requiring that $\sin^2 \theta_{12}^l$ remains within the 3σ range puts an upper bound $\sin^2 \theta_{13}^l \leq 0.0002$ which does not agree with the latest results from T2K and MINOS showing that a small perturbation from type-I term cannot be consistent with data when the type-II term displays exact TBM.

B. Perturbation from the charged lepton mixing

A different class of perturbation to TBM arise when U_l deviates from its $Z_2 \times Z_2$ symmetric form given in Eq. (11). In this case, the neutrino mass matrix has TBM structure but the charged lepton mixing leads to departure from it. This case has been considered in the general context [22] as well as in $SO(10)$ context [11]. Within our approach, we can systematically look at the perturbations which change the values of any one or more angles from the TBM value. For example, we can choose U_l as given in Eq. (15) and look at the quality of fits in this case compared to the exact TBM solution. This choice (corresponding to μ - τ symmetry) leaves θ_{23}^l and θ_{13}^l unchanged but perturbs θ_{12}^l . Alternative possibility is to simultaneously perturb all three mixing angles and look at the quality of fit compared to the exact TBM case. We follow this approach. For this we choose U_l to be a general unitary

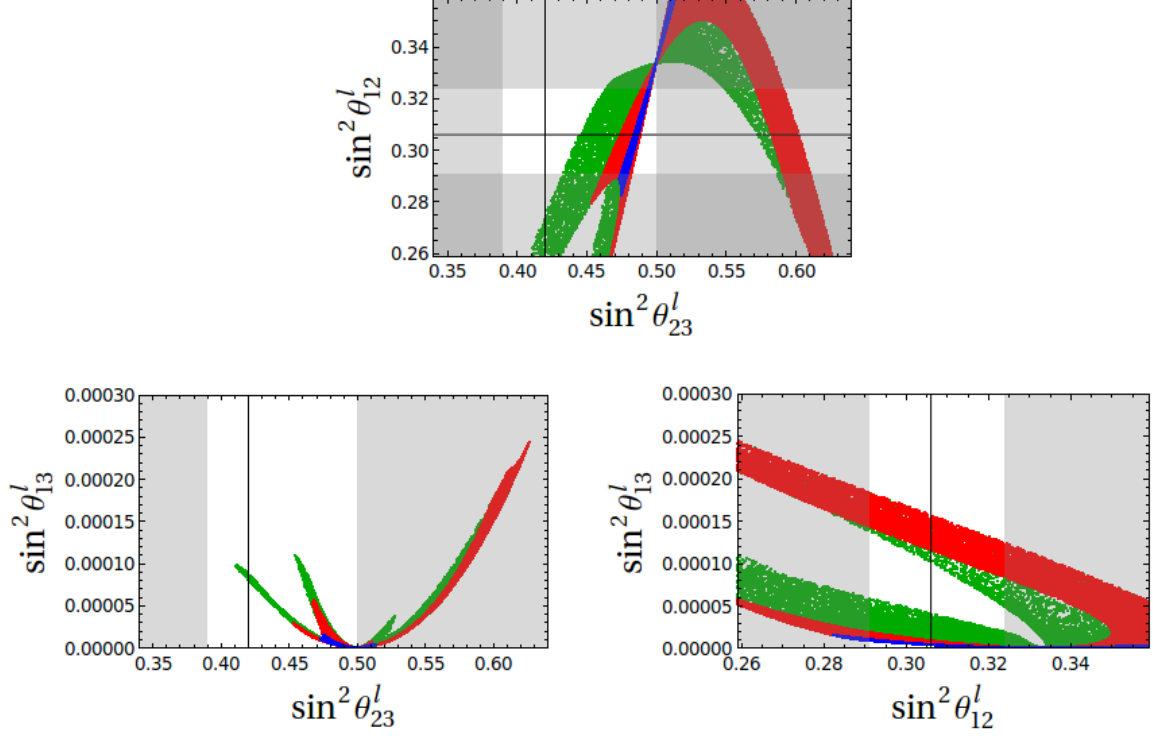


FIG. 1. Correlations among the lepton mixing angles when two real parameters ξ and t_D are varied randomly. The points with different colors correspond to $|t_D| < 1$ (blue), $1 \leq |t_D| < 5$ (red) and $5 \leq |t_D| < 10$ (green). The black lines are the updated central values of $\sin^2 \theta_{12}^l$ and $\sin^2 \theta_{23}^l$ obtained by the global fits on neutrino oscillation data [3]. The unshaded and the shaded regions correspond to 1σ and 3σ bounds respectively.

matrix and go back to the analysis in Section III-A. There, we have fitted the solar and the atmospheric mixing angles to their low energy values given in Table I. Here, we modify the definition of χ^2 and pin down a specific value o_i of the mixing angles p_i by adding a term

$$\chi_{lm}^2 = \sum_i \left(\frac{p_i - o_i}{0.01 o_i} \right)^2 \quad (23)$$

to χ_q^2 that contains all the observables of the quark sector. Sum in Eq. (23) runs over the three lepton mixing angles. The $\chi^2 = \chi_q^2 + \chi_{lm}^2$ is then numerically minimized to fit the 13 observables determined in terms of 12 real parameters as mentioned in Section III-A. Artificially introduced small errors in Eq. (23) fix the value o_i for p_i at the minimum of the χ^2 . We then look at the quantity

$$\bar{\chi}_{min}^2 \equiv \chi^2|_{min} - \chi_{lm}^2|_{min} \quad (24)$$

which represents the fit to the quark spectrum when the lepton mixing angles p_i are pinned down to values o_i . We repeat such analysis by randomly varying o_i within the allowed 3σ ranges of lepton mixing angles [3]. The results are displayed in Fig. (2).

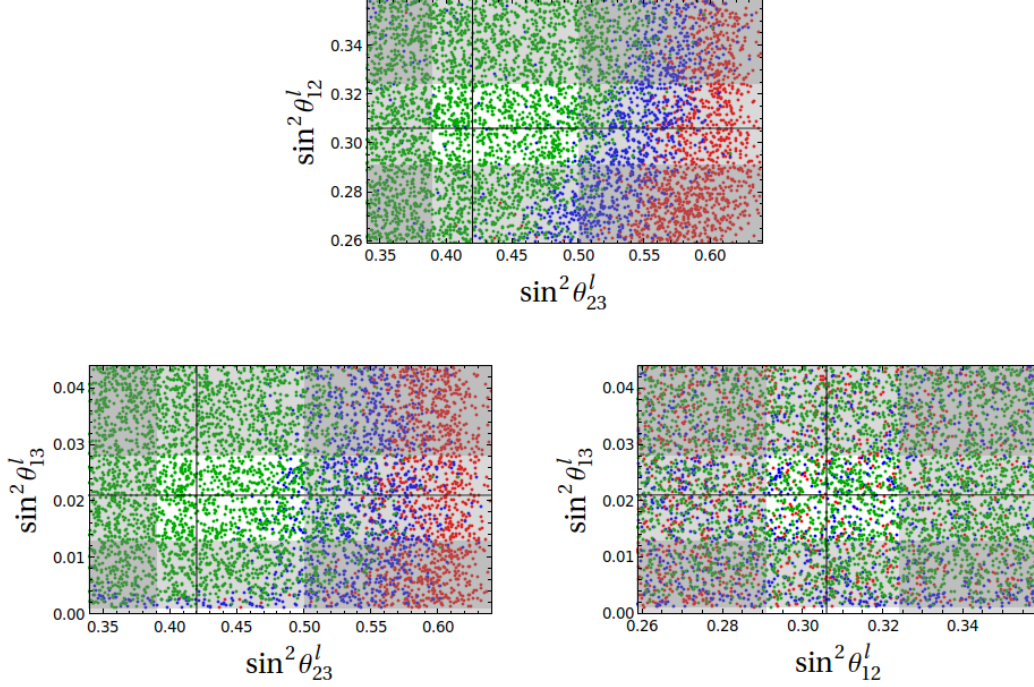


FIG. 2. Correlations among the lepton mixing angles in case of the most general charged lepton mixing matrix U_l . The points with different colors correspond to $\bar{\chi}_{min}^2 < 1$ (green), $1 \leq \bar{\chi}_{min}^2 < 4$ (blue) and $\bar{\chi}_{min}^2 \geq 4$ (red). The black lines are the updated central values of $\sin^2 \theta_{12}^l$ and $\sin^2 \theta_{23}^l$ obtained by the global fits on neutrino oscillation data [3]. The unshaded and the shaded regions correspond to 1σ and 3σ bounds respectively.

We plot the correlations among the lepton mixing angles and show the corresponding values of $\bar{\chi}_{min}^2$ in three different regions. The points corresponding to $\bar{\chi}_{min}^2 < 1$ (green) represent very good fit in which all the observables are fitted within 1σ . The obtained fit shown by the points corresponding to $1 \leq \bar{\chi}_{min}^2 < 4$ (blue) is not as good as the previous one but it is statistically acceptable. The points for $\bar{\chi}_{min}^2 > 4$ (red) represent poor fit and can be ruled out at 95% confidence level. Fig. (2) shows definite correlations between θ_{23}^l and θ_{12}^l . It is seen that the region $\bar{\chi}_{min}^2 < 4$ falls largely below $\sin^2 \theta_{23}^l = 0.58$ for $\sin^2 \theta_{12}^l = 0.306$. It is also seen from the figure that the entire range $0.001 \leq \sin^2 \theta_{13}^l \leq 0.044$ is consistent with statistically acceptable fits to fermion spectrum. This is to be contrasted with the previous case where perturbation from type-I seesaw term led to an upper bound. The bounds obtained numerically allows us to clearly distinguish the case of the exact TBM at M_{GUT} in comparison to the one in which the charged leptons lead to departures from the tri-bimaximality.

V. SUMMARY

The presently available information on the leptonic mixing may be described by a TBM structure for $\mathcal{M}_{\nu f}$ combined with appropriate perturbations generating relatively large θ_{13}^l . We have analyzed the viability of this scenario in a larger context of the grand unified $SO(10)$ theory taking a specific model as an example. The TBM structure for the neutrino mass matrix is a matter of choice of the basis [11]. Thus the existence of TBM is linked to the structure of the charged lepton mixing matrix U_l in this basis. All the related studies in this context [11, 22] assume that U_l deviates slightly from identity and discuss the breaking of the TBM pattern through such U_l . We have shown that it is possible to construct a class of non-trivial U_l quite different from identity which preserve the TBM structure of M_ν when transformed to the flavour basis. Identification of such non-trivial U_l becomes crucial in the context of $SO(10)$ and allows us to obtain a viable fit to fermion spectrum keeping TBM intact. The quality of fit obtained in this case is excellent as shown in Table II and differs only marginally from a general situation without imposing the TBM structure at the outset.

The existence of TBM at the GUT scale may be inferred by considering its breaking which can arise in the model and the reactor mixing angle is a good pointer to this. The quantum corrections are known [21] to lead to very small θ_{13}^l for the hierarchical neutrinos. Similarly, corrections coming from type-I seesaw term imply an upper bound, $\sin^2 \theta_{13}^l \leq 0.0002$ as discussed in Section IV-B. These two cases can be ruled out by relatively large value of θ_{13}^l as indicated by the observations from T2K and MINOS. These cases are in sharp contrast to a situation in which one does not impose the TBM at M_{GUT} and determined U_l from a detailed fits to fermion masses. We have analyzed this scenario using the results from the latest [3] global fits to neutrino oscillation data. It is found that the entire range $0.001 \leq \sin^2 \theta_{13}^l \leq 0.044$ is consistent with the detailed description of all the fermion masses and mixing angles.

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VI. APPENDIX

In this Appendix, we show the parameter values for each of the two cases (A) and (B1) in Table II. The given values of the matrices H , F , G and parameters r , s , t_l and t_u are at the minimum of χ^2 and obtained from our fitting procedure. All the physical mass matrices can be constructed from the given numbers using the $SO(10)$ relations in Eq. (16).

Case A: The most general V_l

The parameter values corresponding to the best fit solution shown in column A in Table II are the following.

$$\begin{aligned}
H &= \begin{pmatrix} 0.00286198 & 0.00178975 & -0.0399313 \\ 0.00178975 & 0.0196685 & 0.00741417 \\ -0.0399313 & 0.00741417 & 1.10192 \end{pmatrix} \text{ GeV} \\
F &= \begin{pmatrix} -0.00235758 & -0.0053869 & -0.0053869 \\ -0.0053869 & -0.0394516 & 0.0317071 \\ -0.0053869 & 0.0317071 & -0.0394516 \end{pmatrix} \text{ GeV} \\
G &= \begin{pmatrix} 0 & -0.00144676 & -0.00442083 \\ 0.00144676 & 0 & -0.0183052 \\ 0.00442083 & 0.0183052 & 0 \end{pmatrix} \text{ GeV} \\
(r, s, t_u, t_l) &= (76.2076, 0.54487, -0.870094, -14.5065)
\end{aligned}$$

Case B: Exact TBM lepton mixing ($V_l = U_l$)

The parameter values corresponding to the best fit solution shown in column B1 in Table II are the following.

$$\begin{aligned}
H &= \begin{pmatrix} 0.00111546 & 0.000428794 & -0.020097 \\ 0.000428794 & 0.0188148 & 0.0639684 \\ -0.020097 & 0.0639684 & 1.17731 \end{pmatrix} \text{ GeV} \\
F &= \begin{pmatrix} -0.0016484 & -0.00375321 & -0.00375321 \\ -0.00375321 & -0.0276748 & 0.0222732 \\ -0.00375321 & 0.0222732 & -0.0276748 \end{pmatrix} \text{ GeV} \\
G &= \begin{pmatrix} 0 & -0.00278749 & -0.0378996 \\ 0.00278749 & 0 & -0.0820076 \\ 0.0378996 & 0.0820076 & 0 \end{pmatrix} \text{ GeV} \\
(r, s, t_u, t_l) &= (70.5742, 0.526782, 0.551405, 2.15986)
\end{aligned}$$

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